Soft multiple parton interactions as seen in multiplicity distributions at Tevatron and LHC

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Abstract

We analyse the multiplicity distributions of charged particles at Tevatron ($p\bar{p}$) and LHC (pp) energies in the framework of the independent pair parton interactions (IPPI) model. It is shown that the number of soft pair parton interactions (and therefore the density of the partonic medium) is large and increases with energy. The mean multiplicity at each parton interaction grows also with energy. This growth depends on the width of the rapidity window. The similar conclusions are obtained in the multiladder exchange model (QGSM).

1 Introduction

Following Feynman, two colliding strongly-interacting high-energy objects (protons, nuclei etc) are considered as sets of pointlike partons. Parton interactions determine the outcome of collisions with many particles produced. They can be either soft, with small transferred momenta, or hard, at large p_T. Most secondary particles are created in soft processes. Usually their characteristics are described within some phenomenological models with their Monte Carlo (MC) implementations. Particles (or jets) with high transverse momentum are more rare and their production is described by perturbative QCD albeit at additional assumptions.

One of the general and widely debated (for reviews see [1, 2, 3]) characteristics reflecting mostly the properties of soft processes is the multiplicity distribution. At energies of tens of GeV the experimental data are well fitted by the negative binomial distribution (NBD) first introduced in [4]. However at higher energies this fit by a single NBD becomes inadequate. A shoulder structure appears at high multiplicities. It is tempting to ascribe it to multiple interactions of pairs of colliding partons. This was done in several models at the expense of introducing new parameters. We have proposed [5] the most economic model with minimum adjustable parameters which we call the independent pair parton interactions (IPPI) model. In fact, we show that there are only two such parameters. Those are the maximum number of active parton pairs at a given collision energy and the average multiplicity of particles produced in the collision of a single pair. At comparatively low

energies one pair is active and it leads to NBD of produced particles which fits experimental data. To deal with the shoulders at higher energies it is assumed that the number of active parton pairs from colliding particles increases with energy. This is in accordance with increase of gluon densities at low shares of momentum x. Interaction of each additional pair results in NBD for its products with the same parameters as for a single pair since the interactions are independent. One is actually tempted to recognize that this assumption could be valid at asymptotically high energies where energy conservation becomes unimportant and the number of colliding partons is infinitely large. We'll try to use it at energies of Tevatron and LHC.

It is the well known property of NBD easily seen from its generating function (see, e.g., [2]) that the convolutions of NBDs again lead to this distribution (see [5]). NBD is characterized by two parameters - mean multiplicity m and dispersion k. With a maximum number of active pairs (j_{max}) at a given energy added, we are left with these three values. However, we show that one of these parameters, namely the dispersion, can be excluded due to some special property of the IPPI model. Thus, only two of them should be used for fits of particle multiplicity distributions at any energy. To compare, there are seven parameters (including j_{max}) in the multiladder exchange or quark-gluon string model (QGSM) [6] used in [7] which are to be fitted by additional energy dependent sets of experimental data on total and elastic scattering cross sections. Even the simple fit with two NBDs asks for six parameters to be used if there are no additional constraints.

Therefore, the IPPI model is the most economic one concerning the adjustable parameters and can serve as a first approximation for estimates of the global characteristics of multiple parton interactions. The IPPI model does not imply that there are no correlations between particles. They are intrinsic in each binary collision because of assumed NBD and in their convolution. Surely, further correlations between these interacting pairs of partons, of both dynamical and kinematical origin, can be introduced as it is done, for example, in the multiladder exchange model [6] which asks for new parameters. Also, there are more rare processes where some pairs interact strongly and are scattered at large transverse momenta (see,e.g., [8, 9]). That would lead to 2-, 4- and more-jets (or $b\bar{b}, \gamma$ etc) production. However, being interesting by itself, the latter aspect is out of reach of the present study.

Let us stress that the knowledge of the number of active parton pairs provides information about the density of the hadronic matter formed in collisions and about its evolution with energy. We shall see that this density is high and grows with energy increase. These findings favor the enlarged role of collective effects at LHC energies.

j_{max}	IPPI		QGSM(14TeV)			
	6	7	7	8	9	10
w_1	0.5041383	0.5020171	0.4272531	0.4237175	0.4222109	0.4216202
w_2	0.2541554	0.2520211	0.2238815	0.2220288	0.2212393	0.2209298
w_3	0.1281295	0.1265189	0.1441260	0.1429333	0.1424251	0.1422259
w_4	0.0645950	0.0635146	0.0945266	0.0937443	0.0934110	0.0932803
w_5	0.0325648	0.0318854	0.0588581	0.0583711	0.0581635	0.0580822
w_6	0.0164172	0.0160070	0.0337454	0.0334661	0.0333471	0.0333005
w_7		0.0080358	0.0176093	0.0174636	0.0174015	0.0173771
w_8				0.0082753	0.0082459	0.0082344
w_9					0.0035557	0.0035507
w_{10}						0.0013988

2 Application of the IPPI model

The main equation of the IPPI model obtained in [5] is

$$P(n; m, k) = \sum_{j=1}^{j_{max}} w_j P_{NBD}(n; jm, jk). \qquad (1)$$

It states that the probability of the n-particle production channel is defined by the sum of NBDs with shifted maxima (jm) and larger widths (jk) for independent parton collisions weighted by their probabilities w_j. The total number of particles n is equal to the sum of particles produced in all active pairs collisions. At asymptotically high energies the probability for j pairs of independent interactions w_j is the product of j probabilities for one pair so that the normalization condition

$$\sum_{j=1}^{j_{max}} w_j = \sum_{j=1}^{j_{max}} w_1^j = 1 \qquad (2)$$

determines w_1 if j_{max} is known at a given energy. In fact, the value of w_1 ranges between 1 at low energies (for $j_{max} = 1$) and 0.5 at asymptotics where j_{max} tends to infinity. Thus, all values w_j are calculated from Eq. (2) if j_{max} is defined. We show them in Table I for 6-10 active pairs which happen to be important at TeV energies.

The IPPI model predicts new special features of moments of the multiplicity distribution which impose some constraints on the parameters and allow to get rid of one of them, namely k. The factorial moments of the

distribution (1) are

$$F_q = \sum_n P(n)n(n-1)...(n-q+1) = \sum_{j=1}^{j_{max}} w_j \frac{\Gamma(jk+q)}{\Gamma(jk)} \left(\frac{m}{k}\right)^q = f_q(k)\left(\frac{m}{k}\right)^q \tag{3}$$

with

$$f_q(k) = \sum_{j=1}^{j_{max}} w_j \frac{\Gamma(jk+q)}{\Gamma(jk)} = k \sum_{j=1}^{j_{max}} w_j j(jk+1)...(jk+q-1). \tag{4}$$

Herefrom one gets the relation

$$m = k \left(\frac{F_q}{f_q(k)}\right)^{1/q}. \tag{5}$$

This relation states that the righthand side with the definite ratio of q-dependent functions should be independent of q for some value of k. It opens the way to the combined fit of experimental mutliplicity distributions with the requirement of q-independence of m (see Appendix 1).

This requirement is very strong and can be satisfied at some special values of w_j unknown to us. It would be too naive to expect it to be precisely satisfied in a simplified model. Nevertheless, we try to find such fits of experimental multiplicity distributions by the IPPI model which minimize the decline of m as a function of q from constancy. After doing this, we get the important information about the maximum number of active parton pairs at a given energy and its evolution with energy. It provides the clear insight into the dynamics of soft interactions and properties of the hadronic medium formed during the collision.

Beside the factorial moments F_q (3) we use the cumulants

$$K_q = F_q - \sum_{r=1}^{q-1} \frac{(q-1)!}{r!(q-r-1)!} K_{q-r} F_r \tag{6}$$

and their ratios

$$H_q = K_q/F_q \tag{7}$$

which possess some specific oscillating behaviour (see [2, 10, 11]). This is the complementary (and sometimes more sensitive!) approach.

We have used both direct fits of multiplicity distributions and the fits of H_q moments at energies of Tevatron 1.8 TeV (for $p\bar{p}$ interactions) [12] and LHC 0.9, 2.36 and 7 TeV (for pp interactions) [13]. The detailed description of our procedure is presented in Appendix 1. The extrapolation of the obtained results admits predictions at 14 TeV which we also show below.
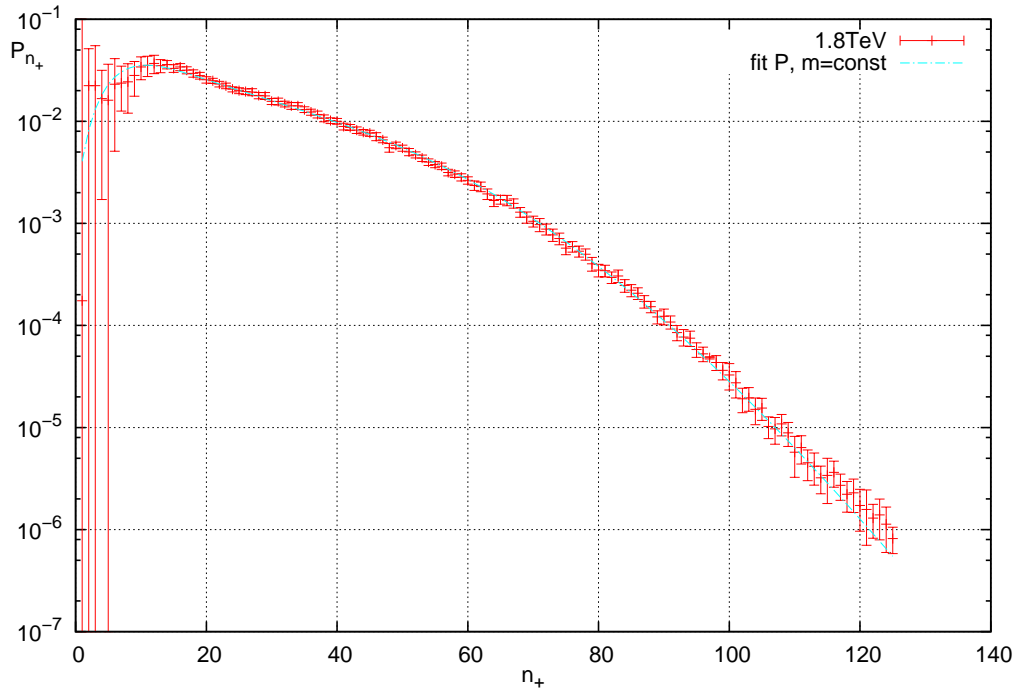
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Figure 1:

The fit of the multiplicity distribution at 1.8 TeV was first done in our paper [5]. Now using the improved procedure described in Appendix 1 we confirmed the stability of previous fits and their parameters as seen from good fit in Fig. 1. The values of fit parameters are $k = 4.42$ and $m = 12.944 \pm 0.04$ which practically coincide with those in [5]. The corresponding fit of H_q is presented in Fig. 2 ($k = 4.36$, $m = 12.90 \pm 0.05$). The values of k and m are close in both approaches. The independence of m on q is satisfied within the limits less than 1 % as seen in Fig. 3. Within these limits the value of k does not play a role and can be considered fixed by constancy of m at a given energy.

The maximum number of the pair parton interactions is $j_{max}(1.8 \text{ TeV}) \approx 4$.

The same procedure was used for the LHC pp-data at 0.9, 2.36 and 7 TeV. The experimental distributions are available for limited pseudorapidity intervals. Namely they have been used by us in distinction to the above fit at Tevatron energy where the data were given for the total pseudorapidity window with some extrapolation to the fragmentation region. Beside some arbitrariness of such extrapolation, the multiplicity distribution for the full phase space is influenced by energy-momentum and charge conservation. These constraints are less important in restricted domains. Therefore, the
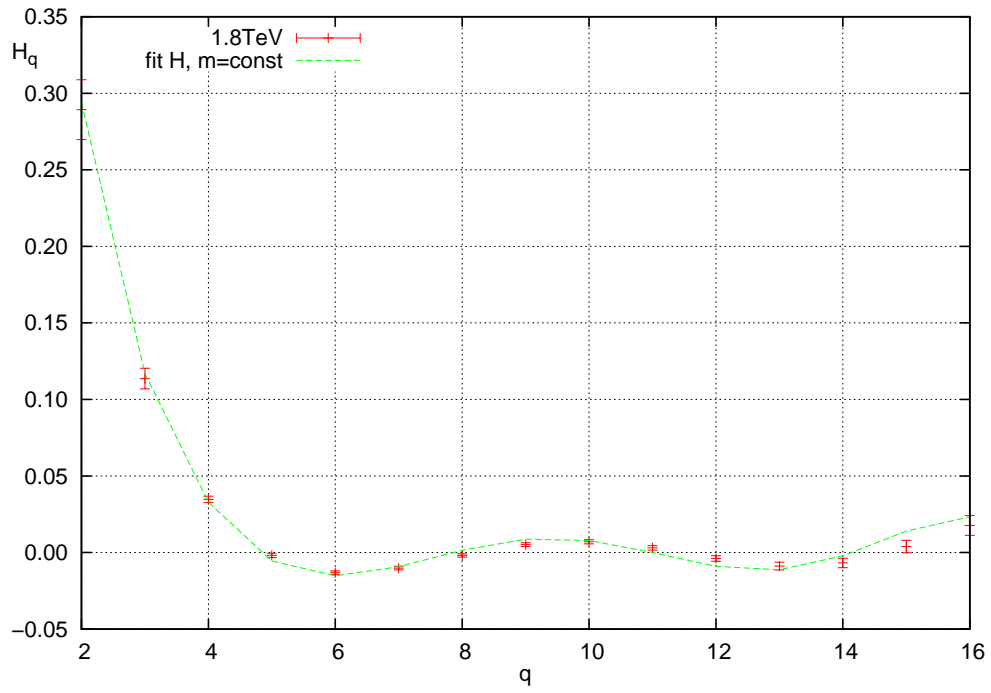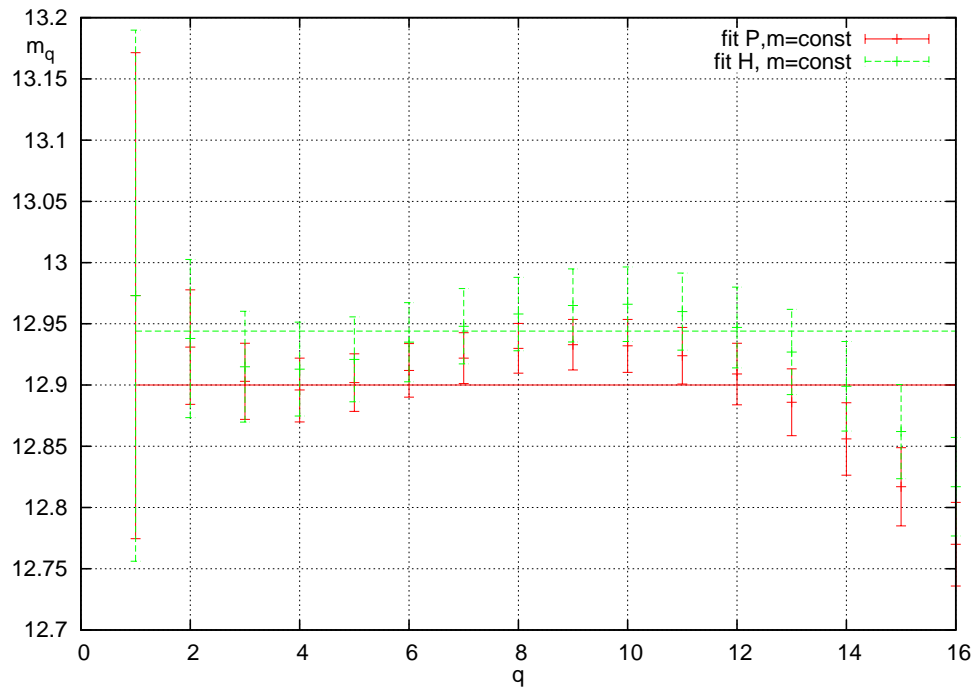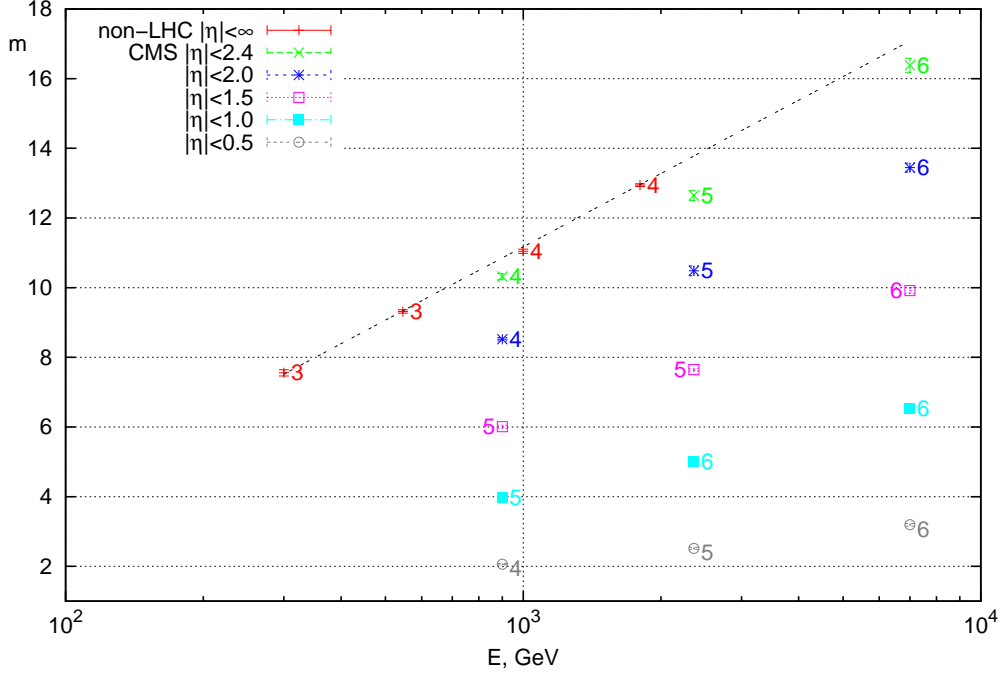
Figure 2:

Figure 3:

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Figure 4:

distributions can be expected to be more sensitive to the underlying dynamics.

The parameter m shows now what part of the multiplicity produced in a single pair parton interaction reaches the analysed pseudorapidity interval. Since intervals $|\eta| < 2.4$ cover the region of almost flat pseudorapidity distribution one would expect that the ratios of corresponding values of m are approximately equal to the ratios of the intervals themselves. On the contrary, in the fragmentation region $|\eta| > 2.4$ the distribution drops down. Thus the values of m for 2.4 must be close to the parameters obtained from extrapolated Tevatron data. All these features as well as the explicit energy dependence of m are clearly seen in Fig. 4 where some results for lower energies from our paper [5] are also shown.

In the same Figure we plot the numbers of active parton pairs from our fits for each pseudorapidity interval near corresponding points. They are quite stable for a fixed energy and rise from 4 at 0.9 TeV to 6 at 7 TeV. The corresponding contributions to the total mean multiplicity (equal to 31.4 at 7 TeV for the interval $|\eta| < 2.4$) of these 6 pairs interactions are 8.3+8.3+6.3+4.2+2.7+1.6.

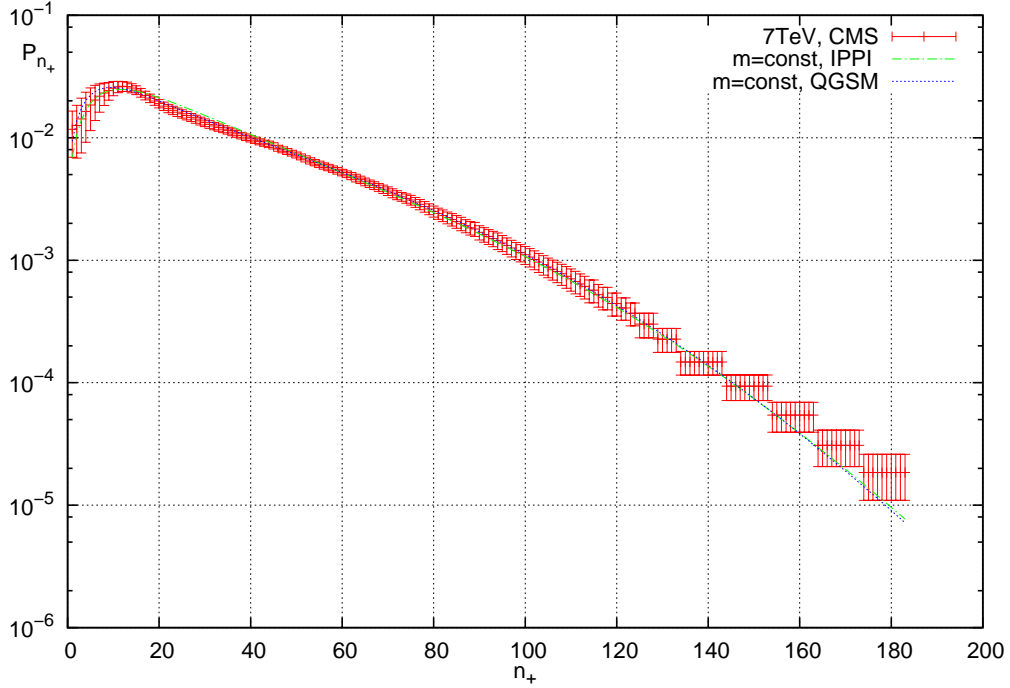The achieved good fits of the shapes of multiplicity distributions with

Figure 5:

above parameters assure that the energy dependence of mean multiplicity and higher moments are also well reproduced.

We have compared our conclusions with those which one would obtain from the multiladder exchange model (QGSM) [6] used in [7]. The lower mean multiplicity of a single pair parton interaction and, correspondingly, somewhat larger numbers of ladders are allowed there due to the wider spread of probabilities w_j. For example, at 7 TeV they would be 13.6 with 8 active pairs of partons. For these fits we have chosen the values of 7 adjustable parameters exactly equal to those used in [7] and can not say how sensitive to their variations are the results. The overall fit of the multiplicity distribution for $|\eta| < 2.4$ according to the multiladder model is somewhat better than in the IPPI model (see Fig. 5). Corresponding χ^2/dof are 62/127 and 131/127.

The qualitative conclusions about the energy increase of the mean multiplicity at a single parton interaction (see Fig. 6) and the number of such interactions (see Fig. 7) are strongly supported in both approaches.

In Fig. 8 we plot the predictions at 14 TeV for $|\eta| < 2.4$ in both models. They are obtained by using the extrapolated values of m shown in Fig. 6. The predicted numbers of active parton pairs at 14 TeV range from 7 in IPPI to 10 in QGSM. In general, we see that the difference between these models
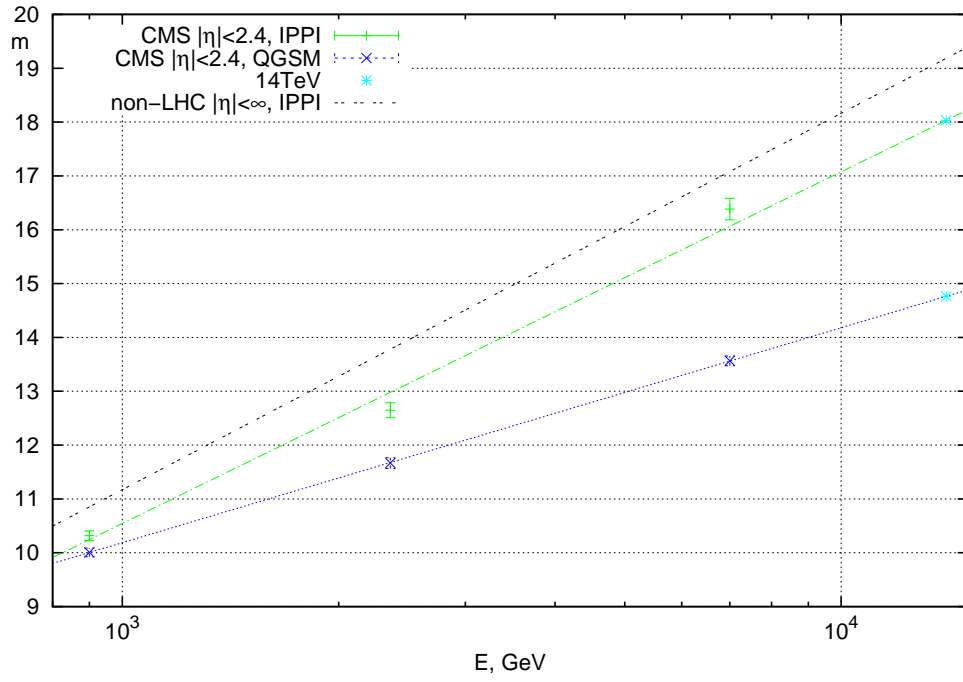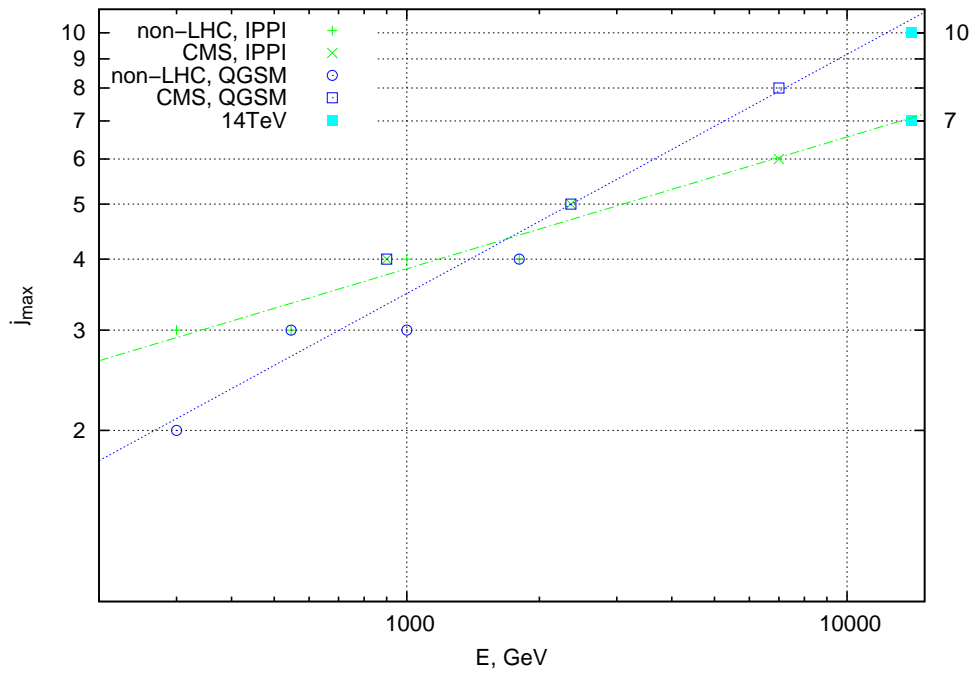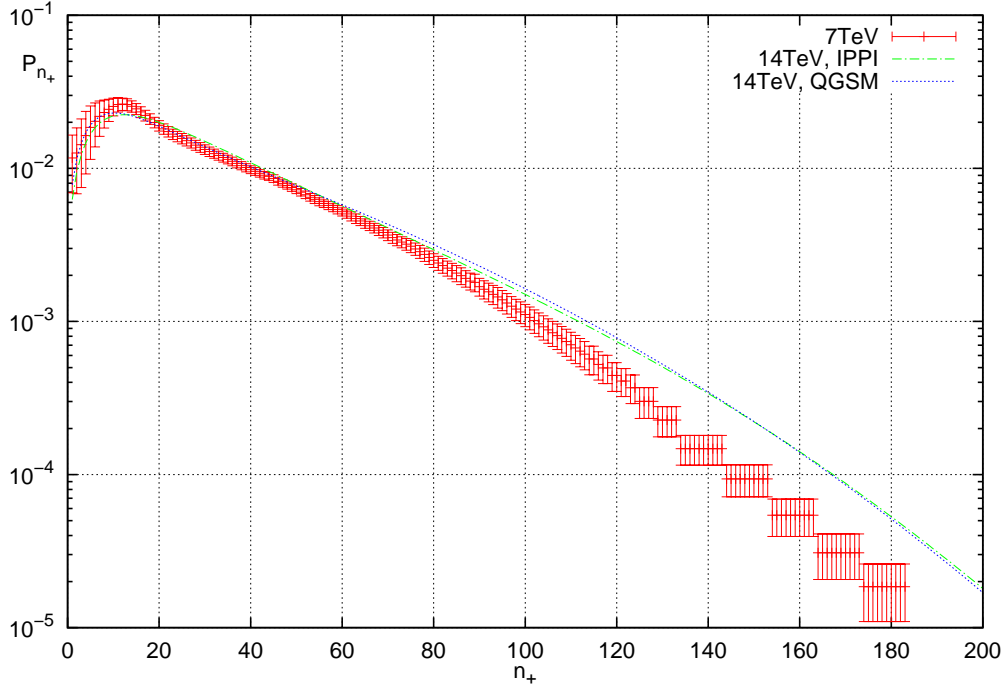
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Figure 6:

Figure 7:

Figure 8:

becomes noticeable only at energies as high as 7 TeV even though it is still not very well pronounced in Fig. 8.

We conclude that the high density partonic medium is formed not only in heavy-ion collisions but also at high energy pp-interactions. The number of active parton pairs taking part in soft pp interactions increases with energy reaching values 6 - 8 at 7 TeV and could be 7 - 10 at 14 TeV, i.e their density is higher at higher energies. Therefore the theoretical account of multiple parton interactions is absolutely necessary at high energies. The mean number of particles created at a single interaction of the pair of partons also increases with energy. This is quite natural because the structure functions are modified correspondingly. The collective effects should become more pronounced at LHC energies.

3 Appendix 1

The experimental data on multiplicity distributions were obtained at Tevatron (for $p\bar{p}$) and LHC (for pp) in various pseudorapidity intervals. Their moments can be computed and compared with predictions of the IPPI model. To get smaller numerical values at the intermediate stages of computing, we

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actually deal with factorial and cumulant moments normalized by $\langle n \rangle^q (q-1)!$.

$$\mathcal{F}_q \equiv \frac{F_q}{\langle n \rangle^q (q-1)!} = \sum_{n=q} P_n \frac{n}{\langle n \rangle} \prod_{r=1}^{q-1} \frac{n/r - 1}{\langle n \rangle}, \tag{8}$$

$$\mathcal{K}_q = \mathcal{F}_q - \sum_{r=1}^{q-1} \frac{\mathcal{F}_r}{r} \mathcal{K}_{q-r}. \tag{9}$$

The values of H_q are not changed by this normalization.

To estimate the errors in moments induced by the experimental error bars of P_n one usually assumes that the latter ones are independent for different n and gets

$$(\triangle F_q)^2 = \sum_n \left(\frac{\partial F_q}{\partial P_n} \right)^2 (\triangle P_n)^2 = \sum_n (n(n-1)\ldots(n-q+1))^2 (\triangle P_n)^2, \tag{10}$$

$$(\triangle H_q)^2 = \sum_n \left(\frac{F_q \frac{\partial K_q}{\partial P_n} - K_q \frac{\partial F_q}{\partial P_n}}{F_q^2} \right)^2 (\triangle P_n)^2, \tag{11}$$

where $\partial K_q / \partial P_n$ is obtained by differentiating (6).

Moreover, one should include the requirement of q-independence of m in the fitting procedure minimizing the decline of values given by Eq. (5) from a constant when calculations are done within some model. Then one minimizes the following functions

$$\min_{k,m} f_P^{Err}(k,m) = \min_{k,m} \sum_n \left(\frac{P_n^{IPPI}(k,m) - P_n}{\triangle P_n} \right)^2 + \sum_{q=2}^{q_{max}} \left(\frac{m(q,k) - m}{\triangle m(q,k)} \right)^2, \tag{12}$$

$$\min_k f_H^{Err}(k) = \min_k \sum_{q=2}^{q_{max}} \left(\frac{H_q^{IPPI}(k) - H_q}{\triangle H_q} \right)^2 + \left(\frac{m(q,k) - \langle m(k) \rangle}{\triangle m(q,k)} \right)^2. \tag{13}$$

Here $m(q,k)$ is computed according to (5) in the IPPI model. As m in Eq. (12) is one of the parameters to be minimized it does not contain experimental errors (in contrast to $\langle m(k) \rangle$ in Eq. (13)) and the denominator in the second sum in Eq. (12) can be computed as

$$\triangle m(q,k) = \frac{m(q,k) \triangle F_q}{q F_q}. \tag{14}$$

The case of Eq. (13) asks for a more complicated formula which is not shown here. Let us note that all the sums are not normalized to the number of particles and the number of the moments since it enlarges the relative

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weight of the constancy of m while our goal is to get the best fit of the multiplicity distributions at the satisfactory fit of additional conditions.

Using these errors one can find out the parameters of the IPPI model in the twofold way: either by the direct fit of multiplicity distributions P_n, or by fits of computed values of H_q. We show the results of both approaches. The latter one is more transparent and the error bars are easily visualized.

There is another problem which becomes important. Namely, one must decide what is the maximum rank (the value of q_{max}) to be used in Eq. (12). To answer this question we introduce a simple criterium related to the experimental error of ΔH_q. We compute the dispersion of ΔH_q according to the standard expression

$$D_q^H = \langle \triangle H^2 \rangle_q - \langle \triangle H \rangle_q^2, \tag{15}$$

where

$$\langle \triangle H^2 \rangle_q = \langle \triangle H^2 \rangle_{q-1} + \frac{1}{q-1}(\triangle H_q^2 - \langle \triangle H^2 \rangle_{q-1}),$$

and

$$\langle \triangle H \rangle_q = \langle \triangle H \rangle_{q-1} + \frac{1}{q-1}(\triangle H_q - \langle \triangle H \rangle_{q-1})$$

with $\langle \triangle H \rangle_1 = \langle \triangle H^2 \rangle_1 = 0$ by definition.

Afterwards we don't consider the moments starting from such rank q that satisfies the condition

$$\frac{\triangle H_q - \langle \triangle H \rangle_q}{\sqrt{D_q^H}} > 1. \tag{16}$$

And in any case we don't consider ranks larger than 16. The maximum number of the pair parton collisions j_{max} is then chosen automatically so that to minimize the value in Eq. (12). In that respect our procedure improves the approach of [5], where this parameter was chosen just among two nearest possibilities. After that one can apply this procedure in attempts to fit experimental data. At 7 TeV we introduce the cut at q_{max}=11 (see Fig.9). Namely these results are described in the main content of the paper.

4   Appendix 2

The q-independence of m is a crucial test of the quality of fits. Let us just briefly mention that we tried to apply the minimization procedure without any condition of constancy of m, i.e. with no last terms in Eqs. (12) and (13). It showed slightly different (albeit within the limits of less than 10 per cents) values of parameters k and m if the Tevatron data at 1.8 TeV
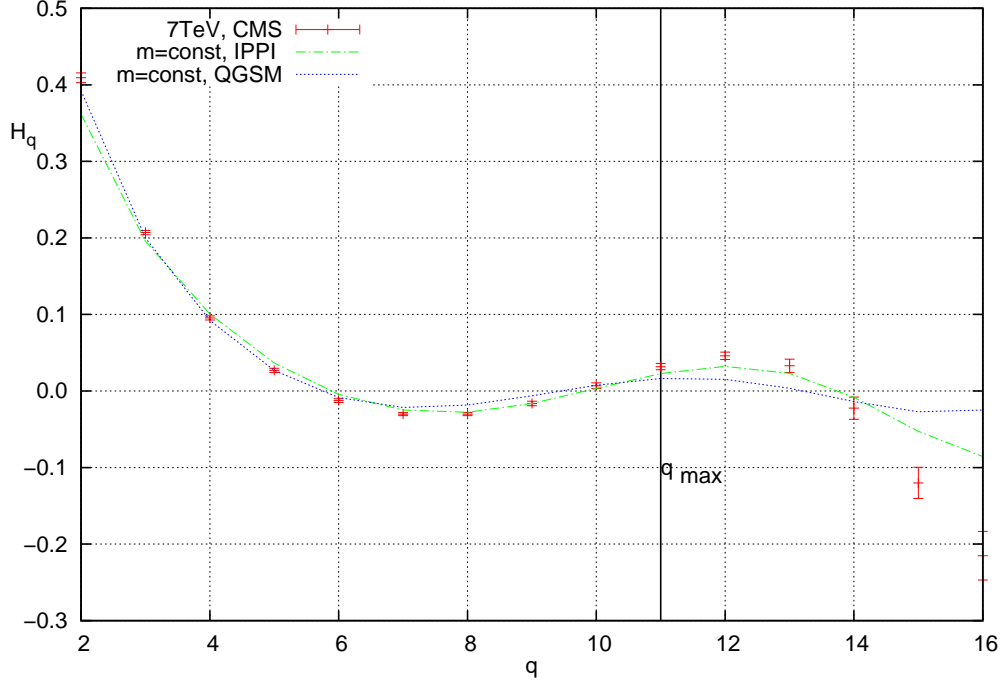
Figure 9:

were used. Visually, these fits looked quite satisfactory (see Fig. 10) both for multiplicity distributions and for H_q. However the conditon $m(q,k)$=const is not well fulfilled. Minimization of probabilities lead to values k=3.79 and m=12.59, while minimization of H_q gives k=4.08 and m=12.828. Fig. 11 demonstrates the q-dependence of $m(q,k)$ (5) for k=3.79 and k=4.08 (the straight lines show the average values). It is much more noticeble than that in Fig. 3. Therefore the last fit discussed in the paper is preferable.

Acknowledgments

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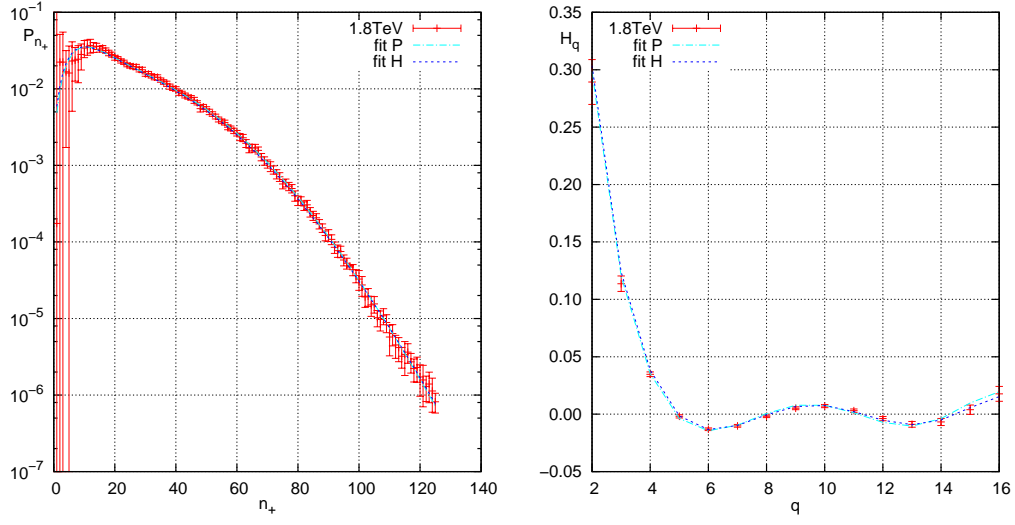[3] J.F. Grosse-Oetringhaus and K. Reygers, J. Phys. G **37**, 083001 (2010).

Figure 10:

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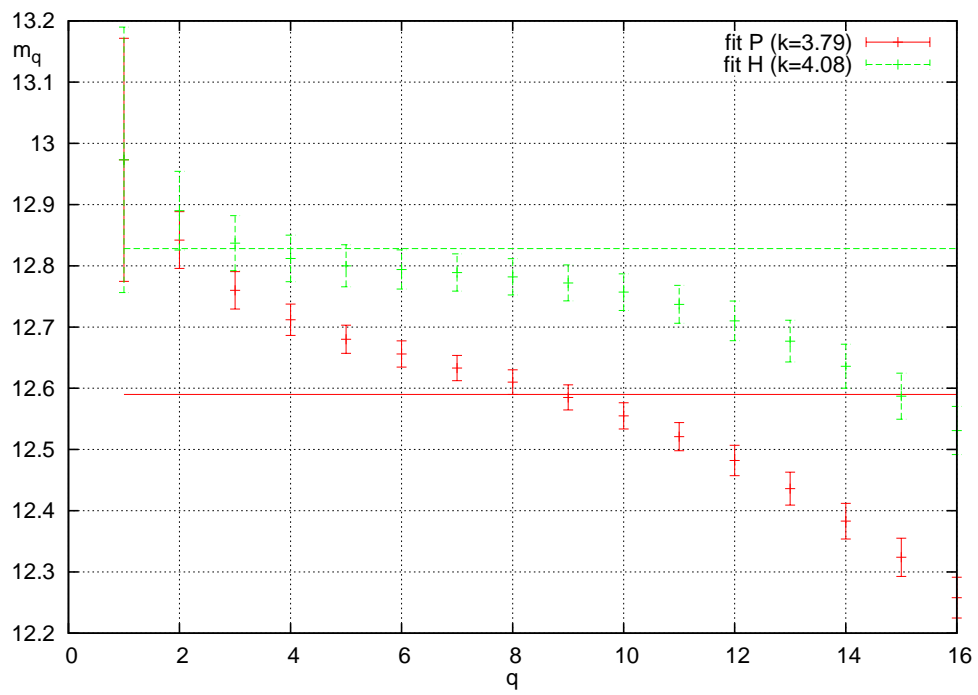[13] CMS Collaboration, V. Khachatryan et al., QCD-10-004, arXiv:1011.5531.

Figure 11: